

The Johns Hopkins University

Magnetic Rotatory Dispersion in Transparent Liquids

DISSERTATION

SUBMITTED TO THE BOARD OF UNIVERSITY STUDIES OF THE JOHNS HOPKINS
UNIVERSITY IN CONFORMITY WITH THE REQUIREMENTS FOR THE
DEGREE OF DOCTOR OF PHILOSOPHY

June 1921



ROBERT ALLEN CASTLEMAN, JR.

Baltimore, Md.

1921

MAGNETIC ROTARY DISPERSION IN TRANSPARENT LIQUIDS

BY R. A. CASTLEMAN, JR., AND E. O. HULBURT

ABSTRACT

Magnetic rotary dispersion of isotropic transparent media.—The electron theory as given by H. A. Lorentz is extended, and a formula for the rotation in a range of spectrum in which electrons of only a single type, with critical frequency c/λ_1 , need be considered, is developed:

$$\theta = \pi(C_1 h_1) \cdot Hl \cdot \frac{[1 + \sigma(\mu^2 - 1)]^2}{\mu} \cdot \frac{\lambda_1^4 \lambda^2}{(\lambda^2 - \lambda_1^2)^2}; \text{ where } C_1 h_1 = \frac{Ne^4}{2\pi^2 cm^2},$$

H is field strength, l is length, μ is refractive index, c is velocity of light, and σ is a constant which Lorentz puts equal to about $\frac{1}{3}$ and Voigt puts equal to zero. To test this theory, substances were chosen whose dispersion was known to conform to the theory of Lorentz, and the magnetic rotations for carbon disulphide, α -monobromnaphthalene, benzene, nitrobenzene and ethyl iodide for six wave-lengths from 436 to 620 $\mu\mu$ were determined with a cell 2 cm long in a field of 6480 gauss. The angles could be measured to $\frac{1}{10}^\circ$ and were found to vary from 4° to 28° , increasing rapidly for each liquid with decreasing wave-length. Theoretical curves were computed, taking the values of C_1 , λ_1 and μ from measurements by others. Below 590 $\mu\mu$ the experimental curves lie below the theoretical curves, the divergence increasing as the wave-length diminishes until the difference at 423 $\mu\mu$ is from 4 to 20 per cent. It is suggested that this discrepancy is due to the absorption of the violet end of the spectrum which was neglected in the theory. The results do not decide between the values 0 and $\frac{1}{3}$ for σ . The values of e/m for the active electrons may be computed from h_1 and vary from 0.5 to 1.78×10^7 e.m.u. according to the liquid and to the value of σ assumed.

1. *Introductory.*—In 1845 Faraday discovered that isotropic substances when placed in a strong longitudinal magnetic field rotate the plane of polarization of plane-polarized light. The obvious explanation of this, confirmed later by Brace, was the same as that offered by Fresnel for the rotation observed in certain crystals, viz., to consider the plane-polarized beam to be composed of two circularly polarized components which travel through the medium with different velocities, one greater and the other less than the velocity of the beam in the medium when unmagnetized. The variation of the angle of magnetic rotation with the wave-length of the light has been considered theoretically by several writers, and the theoretical relations have been applied to the experimental data available. Such data appear, however, to be rather meager, and it seemed desirable to pursue the subject afresh both theoretically and experimentally.

We do not presume to attempt a historical summary of the researches on the many aspects of this subject. For a summary of the earlier work we make reference to an article by P. Joubin.¹ Among the more important theoretical formulae for the magnetic-rotation angle in terms of the wave-length, magnetic field, etc., were those obtained by Joubin, Voigt,² and Drude.³ For a summary of the experimental measurements of the magnetic-rotation angles we turn to the Landoldt-Börnstein *Physical Tables*. There the data have been tabulated in many cases as the *Verdet Constant*, which is defined to be the angle of rotation of plane-polarized light of specified wave-length produced by a medium 1 cm in length when magnetized by a longitudinal magnetic field of strength 1 gauss. These measurements were applied by Joubin and Drude to a dispersion theory of isotropic media, although for such a purpose the experimental data were hardly sufficiently numerous. With the exception of a few transparent liquids and solids the Verdet Constant was found either for unresolved light or for a single wave-length of monochromatic light. In the cases of those substances whose Verdet constant has been determined for a number of wave-lengths throughout the spectrum, one could introduce the values into a suitable dispersion formula to test a dispersion theory. Unfortunately, however, the refractive indices of most of these substances do not conform to any of the simpler dispersion formulae whose constants have a physical interpretation, and so these data are, from this standpoint, uninteresting.

In the present work transparent liquids were chosen whose dispersion conformed approximately to the electronic dispersion theory of H. A. Lorentz.⁴ The angles of magnetic rotation of plane-polarized light for a series of wave-lengths of light in the visible spectrum were measured. A formula of Lorentz giving the relation between the refractive index and the strength of the magnetic field was modified to express the magnetic-rotation angle in terms of the wave-length, the field strength, and other quantities. The formula was found to agree approximately with experiment in

¹ *Thèses*, Faculté des Sciences, Paris, 1888.

² *Magneto- und Elektrooptik*, 1908.

³ *Lehrbuch der Optik*, 1906.

⁴ *Theory of Electrons*, 1909.

those regions of the spectrum wherein the assumptions upon which it was based were valid.

2. *Theoretical*.—Suppose plane-polarized light of wave-length λ *in vacuo* traverses a length l of a medium of refractive index μ for the wave-length in question with a velocity v . If the medium is placed in a magnetic field of strength H , such that the lines of magnetic force are parallel to the direction of propagation of the light, the medium becomes doubly refracting. The two circularly polarized components, which compose the plane-polarized beam, now pass through the medium with velocities v_1 and v_2 , and the medium has the corresponding refractive indices μ_1 and μ_2 , respectively.

Eddy, Morley, and Miller,¹ followed by Mills,² have shown by an interferometer method that

$$\frac{v_1 + v_2}{2} = v, \quad (1)$$

or

$$\frac{1}{\mu_1} + \frac{1}{\mu_2} = \frac{2}{\mu}. \quad (2)$$

This was determined by measurements on carbon disulphide. The accuracy of the work was none too great, because the experiment was a difficult one; but, within the error of observation, the foregoing relation was true. It may be noticed that from general considerations one would not expect (1) to be exactly true, and further that one would expect the difference between (1) and the exact truth to be small. However, we now assume that (1) is true for all the substances and throughout the wave-length range used in this investigation.

Let θ be the observed angle in radians of the rotation of the plane of polarization produced by the magnetized medium. It is easily shown³ that

$$\mu_2 - \mu_1 = \frac{\lambda \theta}{\pi l}, \quad (3)$$

where λ is the wave-length of the light *in vacuo*.

¹ *Physical Review*, 7, 283, 1898.

² *Ibid.*, 18, 65, 1904.

³ Drude, *loc. cit.*, p. 396.

Solving (2) and (3) for μ_1 and μ_2 , and considering $\frac{\lambda\theta}{\pi l}$ small compared with μ , we obtain

$$\left. \begin{aligned} \mu_1 &= \mu - \frac{\lambda\theta}{2\pi l}, \\ \mu_2 &= \mu + \frac{\lambda\theta}{2\pi l}. \end{aligned} \right\} \quad (4)$$

Let us turn to a consideration of the dispersion theory in this connection. We restrict the discussion to isotropic, transparent media in which the temperature remains constant. We use the electron theory of dispersion as given by H. A. Lorentz (*loc. cit.*). Let ξ and E_x be the X components of the displacement of the electron from its equilibrium position and the electric force, respectively. η, ζ, E_y and E_z are the Y and Z components of these quantities; they are all expressed in c.g.s. electromagnetic units. The components of the "restoring force" with which the medium acts upon the electron are $f\xi, f\eta$, and $f\zeta$. The charge on the electron in c.g.s. electromagnetic units is e , its mass is m . N is the number of such electrons per unit volume. σ is a constant which Lorentz has shown to be approximately one-third for isotropic media. The external magnetic field is denoted by H in c.g.s. electromagnetic units. We shall suppose H to have the direction of the axis of Z , which is also the direction of the propagation of the light. The magnetic permeability of the medium is taken as unity.

We find for the equations of motion of the dispersion electron of a single type

$$\left. \begin{aligned} m \frac{d^2\xi}{dt^2} &= e(E_x + 4\pi c^2 \sigma N e \xi) - f\xi + eH \frac{d\eta}{dt}, \\ m \frac{d^2\eta}{dt^2} &= e(E_y + 4\pi c^2 \sigma N e \eta) - f\eta - eH \frac{d\xi}{dt}, \\ m \frac{d^2\zeta}{dt^2} &= e(E_z + 4\pi c^2 \sigma N e \zeta) - f\zeta. \end{aligned} \right\} \quad (5)$$

Let ϵ be the base of natural logarithms, and let all dependent variables of (5) contain the time only in the factor $\epsilon^{i \frac{2\pi c}{\lambda} t}$ where $\frac{c}{\lambda}$

is the frequency, λ the wave-length of the vibration *in vacuo*, and c the velocity of light *in vacuo*. The solution of (5) gives the refractive index μ as determined by the relation

$$\frac{1}{\sigma + \frac{1}{\mu^2 - 1}} = \frac{C_s}{\frac{1}{\lambda_s^2} - \frac{1}{\lambda^2} \pm \frac{H}{\lambda} h_s}, \quad (6)$$

where

$$C_s = \frac{N_s e_s^2}{\pi m_s},$$

$$h_s = \frac{e_s}{2\pi c m_s}.$$

The subscript s is used to denote the s th type of electron. $\frac{c}{\lambda_s}$ is the frequency of the natural undamped vibration of this electron. When the plus sign in equation (6) is used, μ is the μ_2 of (4), and when the minus sign is used μ is the μ_1 of (4).

Equation (6) describes μ in terms of the constants of a single type of dispersion electron. There may be other types of dispersion electrons in the medium with constants peculiar to the type, so that in the more general case the right-hand member of (6) becomes a summation of similar terms, one term for each type. For this case the complete dispersion formula is

$$\frac{1}{\sigma + \frac{1}{\mu^2 - 1}} = \sum \frac{C_s}{\frac{1}{\lambda_s^2} - \frac{1}{\lambda^2} \pm \frac{H}{\lambda} h_s}. \quad (7)$$

We assume we are dealing with a region of the spectrum in which the change of the refractive index with wave-length is determined by the electrons of a single type, so that in the summation of (7) all the terms except one may be replaced by a quantity q_1 which is independent of λ and H . Then (7) becomes

$$\frac{1}{\sigma + \frac{1}{\mu^2 - 1}} = q_1 + \frac{C_1}{\frac{1}{\lambda_1^2} - \frac{1}{\lambda^2} \pm \frac{H}{\lambda} h_1}, \quad (8)$$

where

$$\left. \begin{aligned} C_1 &= \frac{Ne^2}{\pi m}, \\ h_1 &= \frac{e}{2\pi cm}. \end{aligned} \right\} \quad (9)$$

A special case occurs when H is zero. For this (8) becomes equivalent to the well-known dispersion formula of Lorentz:

$$\frac{1}{\sigma + \frac{1}{\mu^2 - 1}} = q_1 + \frac{C_1}{\frac{1}{\lambda_1^2} - \frac{1}{\lambda^2}}. \quad (10)$$

Introducing (4) into (8) gives:

$$\frac{1}{\sigma + \frac{1}{\left(\mu \pm \frac{\lambda\theta}{2\pi l}\right)^2 - 1}} = q_1 + \frac{C_1}{\frac{1}{\lambda_1^2} - \frac{1}{\lambda^2} \pm \frac{H}{\lambda} h_1}. \quad (11)$$

It was expedient to transform this relation to one more amenable to arithmetical computation. This was readily done because $\frac{\lambda\theta}{2\pi l}$ and $\frac{H}{\lambda} h_1$ were smaller than the other quantities appearing with them in the denominators by a different order of magnitude.

Equation (11) becomes to a close approximation

$$\theta = \pi C_1 h_1 H l \frac{[\sigma(\mu^2 - 1) + 1]^2}{\mu} \frac{\lambda_1^2 \lambda^2}{(\lambda^2 - \lambda_1^2)^2}. \quad (12)$$

θ is in radians.

We notice that if we use either the two plus signs of (11) or the two minus signs, we arrive at the same equation for (12). This shows that equation (8) is in close agreement with (4). We notice also that, other quantities remaining constant, θ is proportional to l and H in turn. This agrees with the experimental measurements of Rodger and Watson¹ and Dubois.²

¹ *Phil. Trans. Roy. Soc., A* 186, 621, 1896. This paper contains further references.

² *Wied. Ann.*, 35, 137, 1888.

If σ is given the value $\frac{1}{3}$, equation (12) becomes

$$\theta = \pi C_1 h_1 H l \frac{(\mu^2 + 2)^2}{9\mu} \frac{\lambda_1^4 \lambda^2}{(\lambda^2 - \lambda_1^2)^2}. \quad (13)$$

When σ is placed equal to 0, equation (12) becomes

$$\theta = \frac{\pi C_1 h_1 H l}{\mu} \frac{\lambda_1^4 \lambda^2}{(\lambda^2 - \lambda_1^2)^2}. \quad (14)$$

Formula (14) agrees with one given by Voigt¹ when his quantities are expressed in c.g.s. electromagnetic units.

To compare the experimental measurements with the theory the following procedure was used. Values of the refractive index of the liquid under investigation for wave-lengths in the visible spectrum were taken from data published by others. Three values of μ and the corresponding three values of λ were substituted in equation (10), and the three constants q_1 , C_1 , and λ_1 were computed. Using these three constants, a fourth value of λ was substituted in (10) and the value of μ calculated. If this value of μ agreed approximately with the observed value, the substance was considered to conform to the Lorentz dispersion equation (10) for the region of the spectrum in question. The values of θ for a number of wave-lengths in the visible spectrum were measured for a known length l of the liquid subjected to a known magnetic field H . The constant h_1 was then determined by substituting in equation (13) the values of H , l , C_1 , λ_1 , and the values of λ , θ , and μ for a specified wave-length. θ was then computed by means of (13) for the other wave-lengths, and the computed value was compared with the observed value.

3. *Experimental arrangements.*—The arrangement of the apparatus is shown in plan in Figure 1. A gas-filled lamp with a spiral tungsten filament and a mercury-vapor lamp served as sources of light. The mercury-vapor lamp was used for observations at wave-lengths 435.9 $\mu\mu$ and 546.1 $\mu\mu$; the tungsten lamp was used for observations from the green to the red end of the spectrum.

¹ *Loc. cit.*, p. 130.

The source of light, shown by A , Figure 1, was focused on the slit s_1 of the spectrograph by a lens l_1 , 3 cm in diameter and of focal length 22 cm.

The spectrograph consisted of the Littrow mounting of a plane grating. The grating had a ruled area 6 cm by 7.5 cm and was ruled 15,000 lines to the inch. The cone of light from slit s_1 was reflected by a right-angle glass prism through the large lens l_2 , 10 cm in diameter and with a focal length of 75 cm. The spectrum was brought to a focus at slit s_2 . The grating was mounted on a turntable which could be rotated from the outside of the case containing the spectrograph, so that various wave-lengths of light could be made to pass through the second slit. The grating possessed a bright first order, and this first-order spectrum was

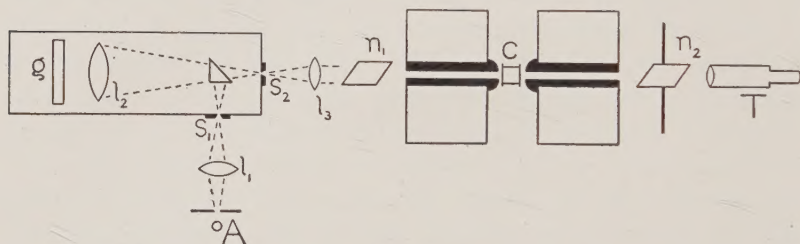


FIG. 1

used throughout the present work. The dispersion was such that with slit s_2 0.25 mm wide a beam of light containing a wave-length range of 5 Å, or $0.5 \mu\mu$, passed through. Both slit s_1 and slit s_2 were always 0.25 mm in width.

The monochromatic beam of light emerging from slit s_2 , after being rendered parallel by a lens l_3 , 2.5 cm in diameter and of focal length 12.5 cm, passed through the polarizing nicol n_1 and through the pierced pole pieces of a Ruhmkorff magnet between which the cell C containing the liquid under investigation was placed. The light then traversed the analyzing nicol prism n_2 and finally entered the telescope T . The lenses l_1 , l_2 , and l_3 were achromatic doublets.

The cell which contained the liquid was made of glass. A short length, about 2 cm, of glass tubing of internal diameter 1 cm, to which had been sealed a small side tube, was ground until the

ends were plane-parallel to within 0.003 cm. Glass plates with optically plane surfaces were cemented to this. Rubber cement, a mixture of lead oxide and glycerine, and LePage's liquid glue, were found useful as cements in various cases. The cell was filled through the side tube. The length of the column of liquid was found by subtracting the thickness of the end plates from the external length of the cell. The liquids used were obtained from the Chemical Laboratory and were of a high degree of purity.

The pole pieces of the Ruhmkorff magnet were elliptical in shape to give a uniform magnetic field; they were adjusted to be 2.5 cm apart. To calibrate the magnet, the angle of rotation θ for sodium light was observed for a known length of carbon disulphide for a current of 26.0 amperes through the magnet. From θ and from the value of the Verdet constant of carbon disulphide for sodium light, which has been carefully determined by Rodger and Watson (*loc. cit.*), the average strength of the magnetic field between the pole pieces was calculated, and was found to be 6480 gauss. This field strength was used for all the rotation measurements of this paper. It was found by tests that the rotation angle θ had the same absolute value for the magnetic field direct and reversed. This indicated that hysteresis effects in the magnet were negligible.

The mounting of the analyzing nicol carried a circular scale which measured the rotation angles to one-tenth of a degree of arc. It was found that the analyzer could be set for extinction for all the wave-lengths with a precision of two-tenths of a degree. The values of the rotation angle were in all cases the means of at least four measurements, two taken with the field direct and two with the field reversed. It was considered that the mean angle was correct to one-tenth of a degree of arc.

4. *Errors and corrections.*—No correction was made for the error due to scattered light. There were two ways in which scattered light might introduce systematic error into the rotation measurement, the first arising from light scattered by the grating, and the second from multiple reflections by the surfaces at the ends of the cell which held the liquid. To determine the effect of the light scattered by the grating, the beam issuing from slit *s*, was examined with a transmission grating. When the tungsten lamp

was used as the source of light, it was found that the beam contained light of foreign wave-lengths of rather feeble intensity; in the case of the mercury lamp this foreign light appeared still weaker. Upon looking into the telescope and setting the analyzer for extinction it was found that the image of slit s_1 did not fade out against an absolutely black background. Very faint light was seen in the field, due no doubt to the light scattered by the grating and to imperfections throughout the optical system. The settings for extinction could, however, be made with precision, and it was deemed that the extraneous light introduced no appreciable systematic errors. Any error due to multiple reflections from the surfaces of the ends of the glass cell was avoided by rotating the cell slightly until these surfaces were not perpendicular to the beam of light. The error which this caused in the determination of l was negligible.

TABLE I

λ	θ
436 $\mu\mu$	1.7°
503	1.3
546	0.9
579	0.85
589	0.85
620	0.8

It had been feared that errors due to temperature would be troublesome, but it was found that these fears were needless. To carry out a complete series of measurements of the rotation angle for five wave-lengths in the visible spectrum required about an hour. During this time the magnet heated up, and the temperature of the liquid in the cell increased. The increase was never more than 3° C. In the case of carbon disulphide the temperature coefficient of the Verdet constant for sodium light has been determined.¹ A three-degree change in temperature changed the Verdet constant by about 0.5 per cent. In the present case it was considered that errors due to temperature changes were for the most part less than 0.5 per cent, and that it was unnecessary to arrange a more accurate control of the temperature of the liquid in the cell.

¹ Rodger and Watson, *loc. cit.*

The rotation angle θ due to the liquid alone was obtained from the observed rotation angle produced by the liquid in the cell by subtracting from this observed angle the angle of rotation produced by the empty cell for the wave-length in question. Table I shows θ for the two glass plates on the ends of the cell. The thickness of the two plates together was 0.315 cm, the field strength was 6480 gauss. The magnetic-rotation angles plotted in the figure have in all cases been corrected for the effect of the glass ends of the cell.

5. *Carbon disulphide*.—The data for this substance and the results of the calculations are given in Table II and Figure 2. We shall discuss these in some detail, and shall avoid a repetition of the discussion for the other substances. The observed values of θ , shown by circles in Figure 2, have been plotted as ordinates against wave-lengths as abscissae; a smooth line, curve 1, has been drawn through them. The strength of the magnetic field, the length of the layer of liquid, and the temperature at the beginning and the end of the experiment are shown in the first two lines of Table II.

Verdet¹ has recorded relative values of θ for carbon disulphide for a number of wave-lengths in the visible spectrum. The magnetic field used was not mentioned. For the sake of comparison the values given by Verdet have been reduced to agree with curve 1 for λ 589.3 $\mu\mu$ and are shown by crosses in Figure 2. Joubin (*loc. cit.*) also carried out measurements on carbon disulphide. Neither the magnetic field, nor the length of the liquid, nor the temperature were recorded. By a coincidence his value for θ at λ 589.3 $\mu\mu$ was the same as that of curve 1, namely 10°3. His values have been plotted as dots in Figure 2. We think that the measurements of curve 1 were correct, for they were repeated with precision a few weeks later.

In order to introduce these experimental results into the dispersion formula (10) we assume that the absorption of carbon disulphide is inappreciable for the visible wave-lengths in question. Such an assumption is manifestly not accurate, because this substance absorbs the blue end of the spectrum to a certain extent.

¹ Verdet, *Oeuvres complètes*.

We take the values of the refractive index found at 20° C. by Flatow. These and the corresponding wave-lengths are shown in

the third and fourth lines of Table II.

Substituting these values into formula (10), the values of the constants C_1 , q_1 , λ_1 were computed and are tabulated in the fifth line of Table II.

The agreement between (10) and observations was tested by using the foregoing values of the constants and computing μ for λ 394 $\mu\mu$ to be 1.7043.

The observed value was 1.70226, and

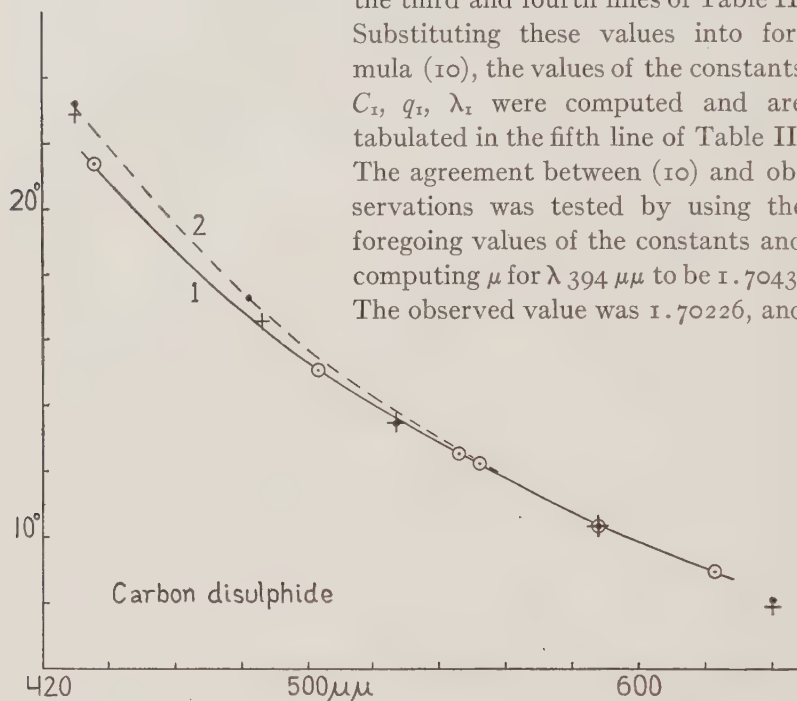


FIG. 2

TABLE II
CARBON DISULPHIDE

$H=6480$ gauss		$l=2.272$ cm	
Temp. $21^{\circ}2$ to $22^{\circ}0$ C.			
λ 441.6 $\mu\mu$	508.6	589.3	
μ 1.67180	1.64586	1.62806	at $20^{\circ}0$ C.
Observed by Flatow, <i>Ann. d. Phys.</i> , 12, 85, 1903.			
$C_1=10.381 \times 10^8$	$q_1=0.57272$	$\lambda_1=204.2 \mu\mu$	
observed $\mu=1.70226$			
λ 394 $\mu\mu$	calculated $\mu=1.7043$		
$h_1=3.95 \times 10^{-5}$ for λ 589.3 $\mu\mu$			

the agreement was considered sufficiently close. These numbers are given in lines 7 and 8 of Table II. The dispersion of carbon disulphide and α -monobromnaphthalene has been more fully discussed in a former paper.¹

The constant h_i was then determined by substituting in equation (12) the values of H , l , C_i , and λ_i given in Table II, and the values of λ , θ , and μ for the wave-length $589.3 \mu\mu$. This gave 3.95×10^{-5} for h_i , as shown in the last line of Table II, when all the quantities were expressed in c.g.s. and c.g.s. electromagnetic units.

The constants of (13) were now completely known, and (13) was then used to compute the values of θ for the range of the spectrum under investigation. The computed curve is shown by the dotted line, curve 2, of Figure 2. It is seen that the agreement between the observed and calculated values of θ was fairly close for the longer wave-lengths, but that for the shorter wave-lengths the theoretical value was greater than the observed value, the difference between the two values increasing as the wave-length decreases. This difference between theory and experiment may be attributed, in part at least, to the neglect of the effect of absorption in the theory. The discrepancy was in the right direction to be attributed to this effect, for the introduction into the theoretical formula of terms denoting absorption will produce a decrease in the computed values of θ for the shorter wave-lengths.

In his treatise on optics Drude² has derived two theoretical expressions for the magnetic rotatory dispersion of isotropic media, one based on the "molecular stream" hypothesis, and the other on the "Hall effect" hypothesis. When written in a form to show the connection between θ and λ , neglecting absorption, the two formulae were, respectively,

$$\theta = \frac{\mu}{\lambda^2} \left(a_1 + \frac{a_2 \lambda^2}{(\lambda^2 - \lambda_i^2)} \right) \quad (15)$$

and

$$\theta = \frac{1}{\mu \lambda^2} \left\{ a_3 + \frac{a_4 \lambda^4}{(\lambda^2 - \lambda_i^2)^2} \right\} . \quad (16)$$

¹ *Astrophysical Journal*, 46, 1, 1917.

² *Loc. cit.*, p. 406.

a_1 , a_2 , a_3 , and a_4 were quantities which involved the masses, the charges, and the numbers of the "bound" and "free" electrons, the strength of the magnetic field, the length of the medium, etc. The refractive index wave-length relation in this connection was

$$\mu^2 = a_5 + \frac{a_6}{\lambda^2 - \lambda_1^2} \quad (17)$$

Drude applied these equations to Verdet's magnetic-rotation measurements on carbon disulphide and creosote in the following manner. Using the value of λ_1 obtained from (17) and two known values each of θ and λ , the remaining two constants of (15) or (16) were determined. The θ - λ curve from either formula, which thus traversed two of the observed points, was found to pass closely to the remaining observed points. We do not believe, however, that the agreement found in this way between theory and experiment possesses great significance. One would not expect a theory which neglected absorption to portray the observations with great exactness.

Joubin (*loc. cit.*) has also derived a formula for the dispersion of magnetic rotation of somewhat the same type as (15). He applied this to the observations of rotations of carbon disulphide and creosote, which were measured for the purpose, in much the same manner as done by Drude.

6. *α -monobromnaphthalene*.—This substance was investigated in a manner similar to that described in the case of carbon disulphide. Table III shows a portion of the data and the results of the calculations. This table has been compiled exactly as was Table II for carbon disulphide, and therefore requires no further explanation. The observed values of θ have been plotted as circles in Figure 3, and a smooth line, curve 1, has been drawn through them. The computed values of θ from equation (13), using the constants of Table III, are shown by the dotted line, curve 2, of Figure 3. The differences between the observed and theoretical values are similar to those noted for carbon disulphide.

7. *Benzene*.—The values of θ for this substance were determined throughout the visible spectrum. These are shown by circles in Figure 4, through which a smooth line, curve 1, has been

passed. The Verdet constant of benzene was found by Jahn¹ to be 0.0297 minutes of arc for sodium light. From the data of Figure 4 and Table IV we obtain 0.0291 minutes of arc for the same wave-length. Jahn's value is not far different from this.

The computed values of θ from equation (13) are shown by the dotted line, curve 2, of Figure 4. It is seen that the differences between the observed and theoretical values are of the same character as those of the previous cases.

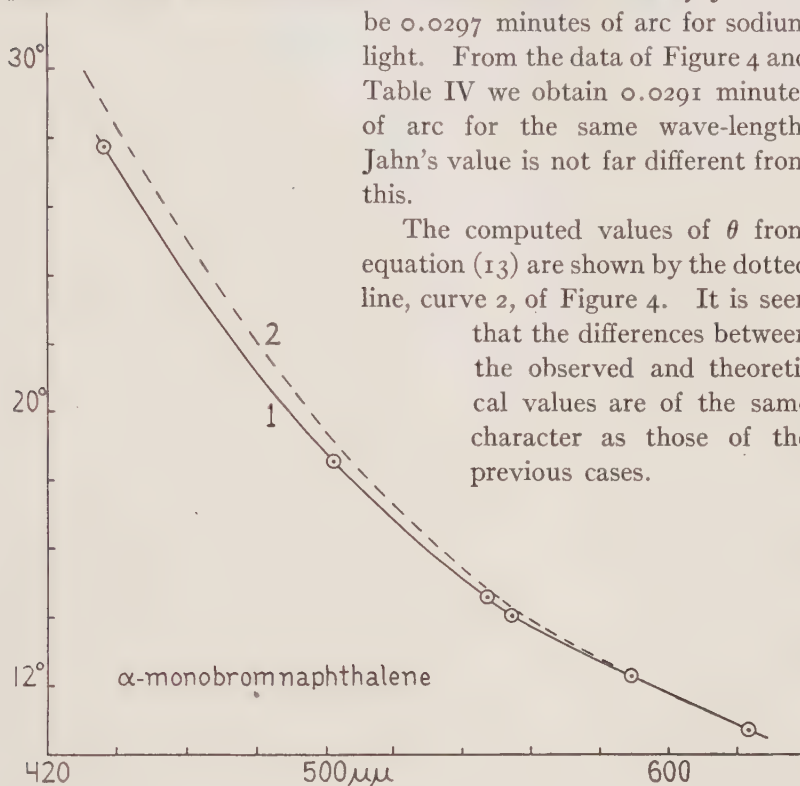


FIG. 3

TABLE III

 α -MONOBROMNAPHTHALENE

$H=6480$ gauss		$l=2.272$ cm	
Temp. 23.5 to 20.7 C.			
$\lambda 434 \mu\mu$	486	589	at 19.4 C.
$\mu 1.70433$	1.68245	1.65876	
Observed by Brühl, <i>Ber. Chem. Ges.</i> , 22, 388, 1897.			
$C_1=7.398 \times 10^8$	$q_1=0.70889$	$\lambda_1=215.6 \mu\mu$	
$\lambda 656 \mu\mu$		observed $\mu=1.64995$	
		calculated $\mu=1.6500$	
$h_1=5.02 \times 10^{-5}$ for $\lambda 589.3 \mu\mu$			

¹ *Wied. Ann.*, 43, 280, 1891.

8. *Nitrobenzene*.—The results of the work on this substance are shown in Table V and Figure 5. The observed values of θ were plotted as circles in Figure 5, and a smooth line, curve 1, has been drawn through them. The computed values of θ from equation (13) are shown by the dotted line, curve 2.

Discrepancies of the same character exist between the observed and theoretical values of θ as were noticed in the preceding cases, but perhaps greater in magnitude.

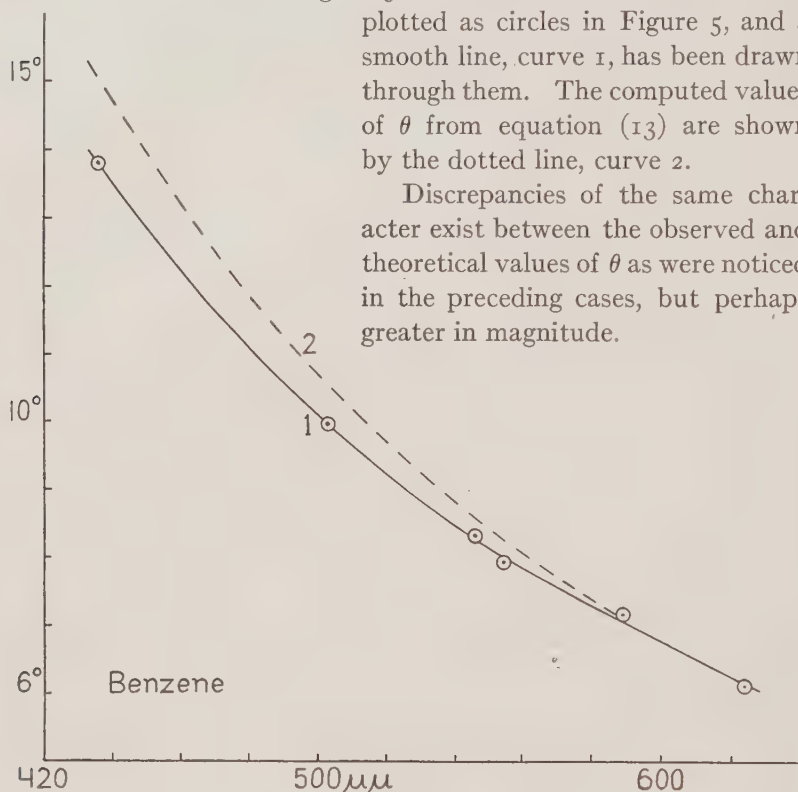


FIG. 4

TABLE IV
BENZENE

$H=6480$ gauss	Temp. 22.9 to 24.6 C.		$l=2.280$ cm
$\lambda\ 434\ \mu\mu$	486	589	
$\mu\ 1.52380$	1.51323	1.50111	at 20.0 C.
Landolt-Börnstein Tables.			
$C_1=11.643\times 10^8$	$q_1=0.40898$	$\lambda_1=173.8\ \mu\mu$	
$\lambda\ 656\ \mu\mu$	observed $\mu=1.49646$		
	calculated $\mu=1.4964$		
$h_1=5.34\times 10^{-5}$ for $\lambda\ 589.3\ \mu\mu$			

9. *Ethyl iodide*.—The circles of Figure 6 show the observations on this substance; a smooth line, curve 1, has been drawn through the observed points. Other data are given in Table VI. Perkin¹ had determined the Verdet constant of ethyl iodide to be 0.0296 minutes of arc for sodium light. From the present data we find 0.0300 for this wavelength. The two values are not greatly at variance. The computed values of θ from equation (12) are shown by the dotted line, curve 2,

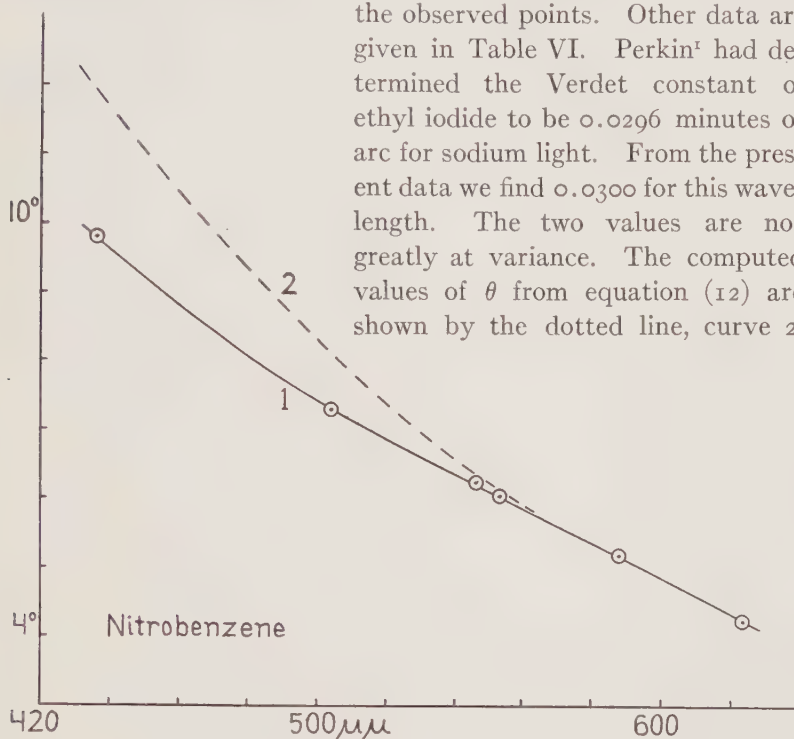


FIG. 5

TABLE V
NITROBENZENE

$H=6480$ gauss		$l=2.272$ cm	
Temp. $21^{\circ}.4$ to $22^{\circ}.6$ C.			
λ 486.2 $\mu\mu$	589.3	656.3	
μ 1.57165	1.55319	1.54641	at $20^{\circ}.0$ C.
Landolt B6rnstein Tables.			
$C_1=6.066\times 10^8$	$q_1=0.62924$	$\lambda_1=217.13$ $\mu\mu$	
λ 434.1 $\mu\mu$		observed $\mu=1.5895$	
		calculated $\mu=1.5888$	
$h_1=2.56\times 10^{-5}$ for λ 589.3 $\mu\mu$			

¹ Smithsonian Physical Tables, 1920.

Figure 6. It is seen that the discrepancies between the observed and theoretical values are similar to those of the preceding cases.

10. *Discussion of results.*—It has been demonstrated that the formula (13), which has been developed from the electron theory of Lorentz, served to express with a certain exactitude the dispersion of the magnetic rotation of certain liquids throughout the visible spectrum. For the longer

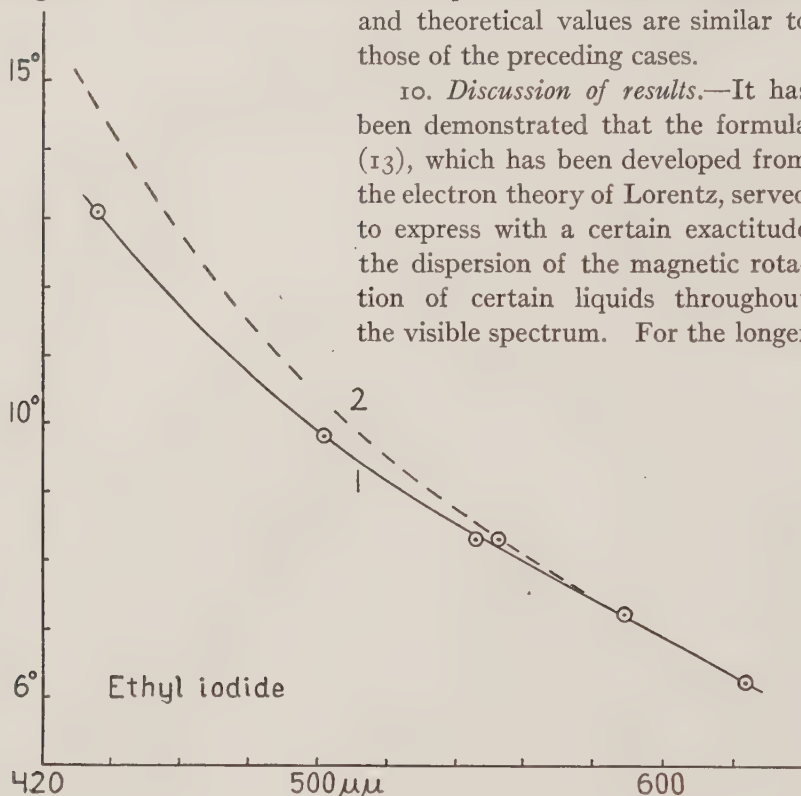


FIG. 6

TABLE VI
ETHYL IODIDE

$H=6480$ gauss	Temp. $22^{\circ}7$ to $24^{\circ}3$ C.		$l=2.220$ cm
$\lambda\ 486.2\ \mu\mu$	589.3	656.3	at $20^{\circ}0$ C.
$\mu\ 1.52356$	1.51203	1.50738	
Observed by Lorentz, <i>Wied. Ann.</i> , 11 , 70, 1880.			
$C_1=12.782\times 10^8$	$q_1=0.50762$	$\lambda_1=168.0\ \mu\mu$	
$\lambda\ 434.1\ \mu\mu$	observed $\mu=1.53437^*$		
	calculated $\mu=1.5336$		
$h_1=5.90\times 10^{-5}$ for $\lambda\ 589.3\ \mu\mu$			

* Haagen, *Pogg. Ann.*, 131, 117, 1866.

wave-lengths of the visible spectrum the agreement between theory and experiment is quite close. For the shorter wave-lengths discrepancies occur which increase as the wave-length decreases. The liquids investigated above possess strong absorption in the ultra-violet and appreciable absorption in the blue end of the spectrum. We may therefore reasonably attribute the discrepancy between theory and experiment in large part to the neglect of absorption. If absorption is taken into account in our equations, λ_1 is increased, and in general the modification which the constants of the equations undergo is such as to bring the theoretical values of θ into closer agreement with the observed ones. The absorption of these substances for light has, however, not been measured accurately, and it seems unprofitable at this time to consider its effect numerically.

11. *The values of e/m .*—From the known value of h_1 , the ratio of the charge to the mass of the electron may be calculated by

TABLE VII

	h_1	e/m
Carbon disulphide.....	3.95×10^{-5}	0.74×10^7
α -monobromnaphthalene....	5.02	0.95
Benzene.....	5.34	1.01
Nitrobenzene.....	2.56	0.49
Ethyl iodide.....	5.90	1.12

means of formula (9). This has been done and the results are shown in Table VII; e/m is expressed in c.g.s. electromagnetic units.

Becquerel¹, Voigt,² and Siertsema³ have derived formulae for the dispersion of the magnetic rotation, by means of which e/m can be found as soon as the Verdet constant and the dispersion $d\mu/d\lambda$ of a substance for the same wave-length λ are known. These three formulae reduce to the same one, namely

$$\frac{e}{m} = \frac{2c}{\lambda} \frac{\theta}{lH} \div \frac{d\mu}{d\lambda}. \quad (18)$$

¹ *Comptes rendus*, 125, 679, 1899.

² *Wied. Ann.*, 67, 351, 1899.

³ *Comm. Lab. Leiden*, No. 82, 1902.

Using (18), Siertsema computed e/m for air, carbon dioxide, hydrogen, water, carbon disulphide, and quartz. The numbers varied from 0.75×10^7 to 1.77×10^7 .

12. *The effect of σ upon the calculations.*—The value of the quantity σ has no very critical effect upon the variation of θ with wave-length. The calculated curves of the diagrams have been obtained by the use of $\frac{1}{3}$ for σ in formula (12). If σ is put equal to 0 in (12), we arrive at Voigt's formula (14), and if this is used to calculate the change of θ with λ we find values of θ which are a trifle less than those obtained from (13). They are about 1 per cent less at λ 434 $\mu\mu$, but are practically the same for wave-lengths greater than 500 $\mu\mu$. If σ is given values greater than $\frac{1}{3}$, the computed values of θ are found to be somewhat greater than the values given by (13), and therefore in greater discordance with the observed values. For example, in the case of carbon disulphide, if $\sigma = \frac{2}{3}$, θ is $25^\circ.1$ at 434 $\mu\mu$.

The values of e/m change to some extent when σ is given different values. This is shown for carbon disulphide in Table VIII. We conclude that, as far as the present data are concerned,

TABLE VIII

σ	e/m
0	1.78×10^7
$\frac{1}{3}$	0.74
$\frac{2}{3}$	0.30
1	0.25

it makes little difference whether σ is 0 or $\frac{1}{3}$. If, however, σ is increased above $\frac{1}{3}$, the discrepancies between the theory and the observations become greater.

In conclusion the authors take pleasure in expressing their thanks to Dr. J. S. Ames for valuable and constructive criticism.

JOHNS HOPKINS UNIVERSITY
February 1921

VITA

Robert Allen Castleman, Jr., son of Robert Allen and Fannie (Funsten) Castleman, was born at The Hague, Westmoreland Co., Virginia, on January 4, 1892. He graduated at the Baltimore City College in 1909, and then attended undergraduate courses at the Johns Hopkins University during the three years, 1910-13. He taught at the Episcopal High School, Alexandria, Virginia, from 1913-15, and took his A.B. degree at George Washington University in 1915. During the year 1915-16 he was assistant in physics at Tulane University, taking graduate courses in physics and mathematics. He returned to Johns Hopkins University in 1916 as a graduate student in physics and electrical engineering, taking his M.A. degree at that institution in 1917. From 1917-20 he was at the Bureau of Standards, holding positions of laboratory assistant and assistant physicist. At the same time he continued his studies under the direction of Professor Ames. He returned to Johns Hopkins in 1920, taking work in physics, mathematics, and applied mathematics. He has had lectures under Professors Ames, Wood, Pfund, Bliss, Whitehead, Cohen, and Murnaghan.

